A theoretical study on the constriction resistance in dropwise condensation

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Abstract—The effect of the thermal conductivity of the condenser material on dropwise condensation heat transfer is studied theoretically. By taking account of the contribution of the droplet resistance in the individual drop size class to the thermal resistance in transient dropwise condensation, a fundamental differential equation describing the constriction resistance caused by the inhomogeneity of surface heat flux is derived. It is found from the non-dimensionalized fundamental equation that the constriction resistance can be determined by a Biot number defined by the interfacial heat transfer coefficient, the departing drop radius and the surface thermal conductivity, in addition to a few characteristic parameters. By applying the so-called equilibrium region of small drops as the drop size distribution, this equation is solved numerically so that the effects of these parameters on the heat transfer coefficient are presented.

1. INTRODUCTION

ALTHOUGH considerable progress has been made in the study on the steam-side mechanism of dropwise condensation, the effects of the thermal properties of the condenser material on the heat transfer rate are less well understood. In dropwise condensation, a very wide range of drop size exists on the condensing surface, extending from the primary drop to the largest departing drop. The condensing surface covered by the smaller active drops yields an extremely high heat transfer rate, while the surface under the larger drops serves as an insulated surface. The nonuniformity of surface heat flux leads to the constriction of heat flow lines near the condensing surface and increases the thermal resistance in the same manner as a contact resistance between the solids. The additional thermal resistance is termed as a constriction resistance and its fundamental explanation was first given by Mikic [1]. The constriction resistance comes up as an important problem for a hydrophobic polymer coating with low thermal conductivity as well as for a practical condenser material.

Experimental evidence for the constriction resistance has been presented by several investigators. Tanner *et al.* [2] compared the heat transfer coefficients in the dropwise condensation of steam at atmospheric pressure using copper and stainless steel surfaces. Griffith and Lee [3] conducted the experiments with the horizontal downward surfaces made of copper, zinc, and stainless steel. Also, Wilkins and Bromley [4] measured the heat transfer coefficients on five kinds of vertical pipes. The above investigators pointed out that the experimental heat transfer coefficient decreased with the surface thermal conductivity. On the other hand, Aksan and Rose [5] measured the heat transfer coefficients on copper and mild steel surfaces very carefully, and they obtained an opposing result that there was no significant difference between them. Most of these workers obtained the condensing surface temperature by the extrapolation method using the temperature profile in the condenser block. Since the uncertainty in inferring the surface temperature from the extrapolation increases with decreasing surface thermal conductivity, many arguments have been made concerning the uncertainty of the experimental heat transfer coefficient together with some discussions on the effect of surface chemistry. Following them, Hannemann and Mikic [6] conducted the precise measurement of the surface temperature using a thin-film resistance thermometer deposited on the stainless steel surface, and they obtained the lower heat transfer coefficient than that for the copper condensing surface. At the same time, Hannemann and Mikic [7] made a numerical analysis which indicated the dependence of the heat transfer coefficient on the thermal conductivity of the condenser material. However, Stylianou and Rose [8] gave once more an opposing experimental result using copper, bronze, and ptfe surfaces. Therefore, it is impossible to conclude from the above studies whether or not the constriction resistance plays an important role in dropwise condensation heat transfer.

This paper puts forward the constriction resistance theory on the basis of numerical work by Hannemann and Mikic [7]. A fundamental differential equation

NOMENCLATURE

area	r _{eri}	th
Biot number, $h_i \cdot r_{\max} / \lambda_c$	$r_{\rm max}$	de
r) fraction of the area covered by drops with	\hat{r}_{\max}	in
radii smaller than r		ra
heat transfer coefficient	r_{\pm}	ch
latent heat of vaporization		eq
interfacial heat transfer coefficient	$T_{\rm c}$	lo
mean free path	$\overline{T_c}$	m
density of drop size distribution	$T_{\rm cm}$	W
mean heat flux		de
local heat flux	T_{s}	sa
r) thermal resistance between the vapor and	t	tir
the condenser surface covered by	U_{g}	sp
drops with radii smaller than r	-	
constriction resistance	Greek s	ymł
dr constriction resistance caused by drops	χ	co
with radii between r and $r + dr$	λ	th
thermal resistance of a single drop		m
gas constant	λ_{i}	th
thermal resistance in the absence of	č	di
constriction effect	ξ_{cri}	di
instantaneous thermal resistance in		r _{cr}
transient dropwise condensation	ξī	di
time-averaged thermal resistance		r_{\pm}
dimensionless thermal resistance, $R \cdot h_i$	τ	di
	area Biot number, $h_i \cdot r_{max}/\lambda_c$ r) fraction of the area covered by drops with radii smaller than r heat transfer coefficient latent heat of vaporization interfacial heat transfer coefficient mean free path density of drop size distribution mean heat flux local heat flux local heat flux r) thermal resistance between the vapor and the condenser surface covered by drops with radii smaller than r constriction resistance dr constriction resistance caused by drops with radii between r and $r + dr$ thermal resistance of a single drop gas constant thermal resistance in the absence of constriction effect instantaneous thermal resistance dimension lass thermal resistance	area r_{cri} Biot number, $h_i \cdot r_{max}/\lambda_c$ r_{max} P)fraction of the area covered by drops with radii smaller than r \hat{r}_{max} heat transfer coefficient r_1 latent heat of vaporizationinterfacial heat transfer coefficient T_c interfacial heat transfer coefficient T_c mean free path T_c density of drop size distribution T_{cm} mean heat flux r_s local heat flux T_s local heat flux T_s r)thermal resistance between the vapor and the condenser surface covered by drops with radii smaller than r constriction resistanceGreek sdrconstriction resistance caused by drops with radii between r and $r + dr$ thermal resistance of a single drop gas constant λ_c thermal resistance in the absence of constriction effect ξ_c toinstantaneous thermal resistance in transient dropwise condensation time-averaged thermal resistance π_r

describing the constriction resistance is derived by formulating the contribution of the drop resistance in the individual drop size class to the total thermal resistance. The characteristics of the fundamental equation are fully analyzed and the effects of each parameter are shown in a more generalized form than that by the numerical study.

2. THEORETICAL BACKGROUND

Mikic [1] pointed out the effect of constriction resistance as follows. The mean heat flux q through the condensing surface A is expressed with the saturation temperature T_s , the local condensing surface temperature T_c , and the local vapor-to-surface thermal resistance R_{1} as

$$q = \frac{1}{A} \int_{A} \frac{T_{\rm s} - T_{\rm c}}{R_{\rm c}} \,\mathrm{d}A. \tag{1}$$

In an extreme case where the thermal conductivity of the condenser material is infinitely high, T_c is uniform and equal to the mean surface temperature $\overline{T_c}$. Then, q is written as

$$q = \frac{T_{\rm s} - \overline{T_{\rm c}}}{R_0} \tag{2}$$

- eparting drop radius
- stantaneous effective maximum drop dius
- naracteristic drop radius defined by uation (16)
 - cal condensing surface temperature
- ean surface temperature
- eighted mean surface temperature efined by equation (5)
- turation temperature
- me
- ecific volume of vapor.

bols

- ndensation coefficient
- ermal conductivity of condenser aterial
- ermal conductivity of liquid
- mensionless drop radius, r/r_{max}
- mensionless critical drop radius, r_{max}
- mensionless characteristic drop radius, $|r_{\rm max}|$
- mensionless time, t/τ_0
- veeping period.

$$\overline{T}_{\rm c} = \frac{1}{A} \int_{A} T_{\rm c} \, \mathrm{d}A \tag{3}$$

$$\frac{1}{R_0} = \frac{1}{A} \int_A \frac{1}{R_\ell} \, \mathrm{d}A. \tag{4}$$

For a practical condenser material having a finite thermal conductivity, T_c is not uniform but variable depending on the local heat flux. In this case, by introducing a weighted mean surface temperature

$$T_{\rm cm} = \frac{R_0}{A} \int_A \frac{T_c}{R_{\ell}} \,\mathrm{d}A \tag{5}$$

we can express the mean heat flux as

$$q = \frac{T_s - T_{\rm cm}}{R_0}.$$
 (6)

Usually the heat transfer coefficient h for dropwise condensation is defined by

$$h = \frac{q}{T_{\rm s} - T_{\rm c}}.$$
 (7)

Therefore, the thermal resistance of dropwise condensation is expressed as

$$R\left(=\frac{1}{h}\right) = \frac{T_{\rm s} - T_{\rm cm}}{q} + \frac{T_{\rm cm} - \overline{T_{\rm c}}}{q}$$
$$= R_0 + R_{\rm cr}.$$
 (8)

where

From the above equation, we can find that the thermal resistance in dropwise condensation is expressed as a sum of two resistances in series, namely, the surface-averaged thermal resistance R_0 through the droplets and the constriction resistance R_c caused by the non-uniformity of the surface temperature.

A thorough understanding of the constriction resistance phenomena requires knowledge of the drop size distribution on the condensing surface. A comprehensive theory for the drop size distribution has been developed [9, 10] from the viewpoint that the so-called steady dropwise condensation is, in reality, composed of transient dropwise condensation occurring repeatedly on the tracks left by departing drops. From the theory, we can estimate the instantaneous drop size distribution by the following equation for the case that the nucleation-site density is infinitely high:

$$N(r,t) = \frac{0.321}{\pi \hat{r}_{\max}^3} \left(\frac{r}{\hat{r}_{\max}}\right)^{-2.679}$$
(9)

where N(r, t) is the density of drop size distribution defined so that there are N(r, t) dr drops per unit condenser area having radii in the interval [r, r+dr]. The drop radius \hat{r}_{max} is termed as an instantaneous effective maximum drop size, which grows with time t from the start of transient condensation and it can be connected with the departing drop radius r_{max} by

$$\frac{\hat{r}_{\max}}{r_{\max}} = 1.53 \left(\frac{t}{\tau_0}\right)^{1/1.3}$$
 (10)

where τ_0 is the mean sweeping period of drop departure. Equation (9) is applicable to the equilibrium region of small drops [9] which develops in the very wide range of drop sizes: $2D < r < 0.2\hat{r}_{max}$, where *D* is the spacing between the nucleation sites. This theoretical expression has been confirmed to be in good agreement with the experimental drop size distribution [11]. The drop size distribution in the steady dropwise condensation is obtained by averaging the instantaneous distribution density over the time interval τ_0 [12].

3. CONSTRICTION RESISTANCE THEORY

3.1. Derivation of basic equation

In analyzing the effects of spatial and time-wise nonuniformity of the surface heat flux due to the timedependent drop size distribution, it is considered that the transient dropwise condensation starts simultaneously at time t = 0 throughout a considerably large, initially bare surface. Here, we assume that the time response of the surface temperature to the variation of drop size distribution is sufficiently rapid such that the thermal capacity effect can be neglected. From the assumption, the instantaneous thermal resistance in the transient dropwise condensation can be determined using the transient drop size distribution and the quasi-steady heat conduction equation. The thermal resistance in the steady dropwise condensation can be obtained by taking a time-average over a sweeping cycle of falling drops. The assumption is based on the facts that the big drops yielding the large constriction do not change their locations so rapidly and that the temperature variation by the small active drops is much smaller than that by the big drops.

At a certain time t in the transient dropwise condensation, let R(r, t) denote the vapor-to-surface thermal resistance in the fractional surface area f(r, t)covered by the drops having radii smaller than r, as shown in Fig. 1. If the thermal resistance is free of constriction, the droplet resistance R_d per unit area having radii in the interval [r, r+dr] is accompanied with R(r, t) in parallel

$$\frac{f(r+dr,t)}{R_{\rm p}} = \frac{f(r,t)}{R(r,t)} + \frac{f(r+dr,t) - f(r,t)}{R_{\rm d}} \quad (11)$$

where R_p is the parallel resistance. In the practical condenser material, constriction resistance $R'_c dr$ caused by the drops of radii r to r+dr takes part in the thermal resistance in series as equation (8)

$$R(r+dr, t) = R_{\rm p} + R'_{\rm c} dr.$$
 (12)

By eliminating R_p from equations (11) and (12), and by ignoring terms of second and higher orders of dr, we obtain the following fundamental differential equation:

$$\frac{\partial R}{\partial r} - \frac{1}{f} \frac{\partial f}{\partial r} R + \frac{1}{f} \frac{\partial f}{\partial r} \frac{R^2}{R_{\rm d}} = R_{\rm c}'.$$
 (13)

Here, the additional constriction resistance $R'_c dr$ is evaluated by the adiabatic cylinder model as shown in Fig. 2. Since the fractional area covered by drops with radii in [r, r+dr] is small in comparison with f'(r, t), it is reasonable to consider that such large drops are arranged with enough spacing between them. In this model, a large drop with a thermal resistance R_d is in the center of the cylinder, and is surrounded by drops with radii smaller than r forming thermal resistance R(r, t). For simplicity, drops are assumed to be hemispherical. From the calculation



FIG. 1. Typical instantaneous drop size distribution in the transient dropwise condensation.



FIG. 2. Adiabatic cylinder model.

of the surface temperature distribution, $R'_{c} dr$ is obtained as

$$R'_{\rm c} \,\mathrm{d}r = \frac{8r}{3\pi\lambda_{\rm c}} \left(1 - \frac{R}{R_{\rm d}}\right)^2 \left(1 + \frac{8r}{3\pi\lambda_{\rm c}R_{\rm d}}\right)^{-1} \frac{1}{f} \frac{\partial f}{\partial r} \,\mathrm{d}r.$$
(14)

The derivation is given in the Appendix.

The thermal resistance through a hemispherical droplet R_d is expressed from the substantial drop growth rate by Umur and Griffith [13], which is approximated by [14]

$$R_{\rm d} = \frac{1}{2h_{\rm i}(1 - r_{\rm cri}/r)} \frac{3.5r/r_{\rm 1}}{\ln\left(1 + 3.5r/r_{\rm 1}\right)}.$$
 (15)

Here, r_{cri} is the thermodynamic critical drop radius [15], and r_1 the following characteristic drop size at which the interfacial thermal resistance and the conduction resistance through the drop approximately balance [10]:

$$r_1 = \frac{2\lambda_{\gamma}}{h_{\rm i}} \tag{16}$$

where λ_{ℓ} is the thermal conductivity of the liquid. The interfacial heat transfer coefficient h_i is given by [16]

$$h_{\rm i} = \frac{2\alpha}{2 - 0.789\alpha} \frac{1}{\sqrt{(2\pi R_{\rm g} T_{\rm s})}} \frac{h_{\rm fg}^2}{v_{\rm g} T_{\rm s}}$$
(17)

where R_g is the gas constant, v_g the specific volume of the vapor, h_{fg} the latent heat of vaporization and α the condensation coefficient with a value of 0.4 for low pressure steam [12].

Equation (13) is a Riccati differential equation, but it is difficult to obtain an analytical solution except for the two limits of $\lambda_c \rightarrow \infty$ and $\lambda_c \rightarrow 0$. In the former case ($\lambda_c \rightarrow \infty$), i.e. $R_c = 0$, equation (13) has the following solution :

$$\frac{f(r,t)}{R(r,t)} = \int \frac{1}{R_{\rm d}} \frac{\partial f}{\partial r} \,\mathrm{d}r. \tag{18}$$

The solution is also obtained from the physical meaning that the heat transfer coefficient in the absence of constriction resistance is given as a total conductance for all drops. In the latter case $(\lambda_c \rightarrow 0)$, equation (13) has the solution

$$f(r,t)R(r,t) = \int R_{\rm d} \frac{\partial f}{\partial r} \,\mathrm{d}r. \tag{19}$$

Here, let us consider the physical meaning of this equation. If we denote the local heat flux as q_ℓ , the local subcooling is expressed as $R_d \cdot \mathbf{q}_\ell$. Then, the surface averaged subcooling $\Delta T_m(r)$ for the fractional area f(r) is given as

$$\Delta T_{\rm m}(r) = \frac{1}{f} \int R_{\rm d} q_{\nu} \frac{\hat{c} f}{\partial r} \, {\rm d}r.$$
 (20)

Since the thermal resistance at this surface is R(r), the subcooling is also expressed as

$$\Delta T_{\rm m}(r) = \frac{R}{f} \int q_{\ell} \frac{\partial f}{\partial r} \,\mathrm{d}r. \tag{21}$$

Comparing equation (21) with equation (20), we find that equation (19) should hold if q_{ℓ} is constant. In the case of $\lambda_c \rightarrow 0$, the surface temperature is almost the same as the steam temperature T_s since the steam-side heat transfer coefficient is very high compared with the conductance of the condenser material. Therefore, the deviation of temperature gradient in the condenser material from the mean value can be disregarded, so that the local heat flux q_{ℓ} is considered to be constant.

3.2. Basic parameters and numerical method

Taking r_{max} and h_i as characteristic scales for length and thermal conductance, respectively, a dimensionless drop radius ξ and a dimensionless thermal resistance R^* are defined by

$$\xi = r/r_{\text{max}}, \quad R^* = R \cdot h_i. \tag{22}$$

Then, the fundamental equations (13) and (14) are made dimensionless as follows:

$$\frac{\partial R^*}{\partial \xi} = \frac{1}{f} \frac{\partial f}{\partial \xi} (R_{\rm d} - R^*) \frac{R^* + 8/(3\pi) Bi_{\rm c} \xi}{R_{\rm d}^* + 8/(3\pi) Bi_{\rm c} \xi} \quad (23)$$

where Bi_c is a Biot number defined by the thermal conductivity of the condenser material λ_c , interfacial heat transfer coefficient h_i , and departing drop radius r_{max}

$$Bi_{\rm c} = \frac{h_{\rm i} \cdot r_{\rm max}}{\lambda} \,. \tag{24}$$

The droplet resistance in equation (15) is transformed to the non-dimensional form

$$R_{\rm d}^* = \frac{1}{2(1 - \xi_{\rm cri}/\xi)} \frac{3.5\xi/\xi_1}{\ln\left(1 + 3.5\xi/\xi_1\right)}.$$
 (25)

It is found from equations (23) and (25) that the thermal resistance in the dropwise condensation is determined by the Biot number Bi_c in addition to the basic parameters ξ_1 and ξ_{cri} shown by the theory given in ref. [10]. When the condensing substance, system pressure, mean surface subcooling, and departing

drop size are given, the basic parameters ξ_1 and ξ_{cri} are determined. Thus, the effect of surface thermal conductivity on the heat transfer coefficient depends on the value of Bi_c .

The differential equation (13) is solved numerically by the Runge-Kutta method from the drop size ξ_1 until the effective maximum size ξ_{max} at a certain instant of the transient dropwise condensation. Here, the instantaneous drop size distribution in equation (9) is assumed to be applicable for all of the drop size region

$$f(\xi, \tau) = (\xi/\hat{\xi}_{\max})^{0.321}.$$
 (26)

The procedure is repeated by the proper time increment and the time-averaged thermal resistance is obtained using the trapezoidal rule.

In the drop size region $\xi < \xi_1$, the interfacial resistance is more dominant than the conduction resistance through the droplet. The effect of mean free path L on the droplet thermal resistance $R_{\rm d}$ was pointed out in ref. [17]. Noticing that equation (15) assumes uniform h_i over the drop surface, it proves to be valid in the case of $L/r \ll 1$. However, in the present case in which the surface is completely covered by drops, the thermal resistance of the droplets for which $L/r \gg 1$ increases to twice the value of equation (15) [17, 18]. The modification factor $[3 + \log_{10} (L/r)]/4$ was introduced [12] to change the droplet conductance for the drop size region 0.1 < (L/r) < 10. Then, the droplet resistance R_d^* of equation (15) is modified by the multiplier from two to one. From this modification, the thermal resistance R_d^* in the region $\xi < \xi_1$ becomes almost uniform at unity except near ξ_{cri} . Therefore, the initial and boundary conditions in the present calculation are determined as follows:

$$R^* = 1$$
 at $\tau = 0$ and $R^* = 1$ at $\xi \leq \xi_1$.
(27)

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The value of ξ_{cri} is fixed as 2.0×10^{-5} throughout the calculation since r_{cri} does not change markedly.

4. RESULTS AND DISCUSSION

Numerical analysis is first carried out for the case of the transient dropwise condensation of steam at atmospheric pressure under the departing drop size $r_{max} = 1.0$ mm. In this case, ξ_1 takes a value of 3.64×10^{-4} . Figure 3 presents the calculated variation



FIG. 3. Variation of instantaneous thermal resistance with time.



FIG. 4. Relation between time-averaged thermal resistance and Biot number.

with time τ of the instantaneous thermal resistances \hat{R}^* for the various values of Bi_c . The instantaneous resistance increases with time due to the growth of the droplet even in the case of $Bi_c = 0$. The increasing rate of thermal resistance becomes significant with Bi_c . We note from this figure that the contribution of large drops having radii near the departing drops is marked and that an increase of Bi_c raises the constriction resistance in the regions of smaller drops.

Figure 4 shows the time averaged thermal resistance $\overline{R^*}$ as a function of Bi_c . Here, we chose the values of $\xi_1 = 1.29 \times 10^{-2}$, 4.71×10^{-2} , 1.80×10^{-3} , and 3.64×10^{-4} , which correspond to the steam pressures of 1.0, 3.56, 12.3, and 101.3 kPa, respectively. For small Bi_c , the time averaged thermal resistance is constant at the lower limit obtained from equation (18) for $\lambda_c \to \infty$ as

$$\overline{R^*} = 0.629\xi_1^{-0.3}.$$
 (28)

The thermal resistance increases with Bi_c and gradually approaches the upper limit for $\lambda_c \rightarrow 0$, which is expressed from equation (19) as

$$\overline{R^*} = 0.177\xi_1^{-0.82}.$$
(29)

It is also found that the increasing rate of resistance is marked for the small values of ξ_1 . This is because the drop size region effective for the constriction resistance extends to the smaller drops due to relatively large droplet conduction resistance compared with the interfacial thermal resistance of higher pressure steam.

The heat transfer coefficients are presented in Fig. 5 for two steam pressures of 1.0 and 101.3 kPa with a practical purpose of illustrating how the heat transfer coefficient decreases with the decrease of surface thermal conductivity. The theoretical predictions from the present constriction resistance theory are shown by the solid and broken lines for the radii of departing drops $r_{\text{max}} = 1.0$ and 1.5 mm, respectively. For a surface material with a high thermal conductivity such as copper ($\lambda_c \sim 380 \text{ W m}^{-1} \text{ K}^{-1}$), there is no significant effect of the constriction resistance at the steam pressure of 1.0 kPa, but the heat transfer coefficients at 101.3 kPa decrease to about 80% of the theoretical results for a surface with infinite thermal conductivity. Further, for the case of a glass surface with very low thermal conductivity ($\lambda_c \sim 1.0 \text{ W} \text{ m}^{-1} \text{ K}^{-1}$), the



FIG. 5. Variation of heat transfer coefficient for dropwise condensation of steam with surface thermal conductivity.

effects of constriction resistance are considerably large such that the heat transfer coefficients decrease to onefifteenth at 101.3 kPa and one-third at 1.0 kPa. Figure 5 also shows the representative heat transfer data by some researchers for comparison with the present results. The theoretical predictions for the copper surface at 1.0 kPa agree very well with the reliable data of ref. [19] while theoretical lines for 101.3 kPa deviate to some extent to the higher side. The heat transfer coefficients for the atmospheric pressure steam depend on the nucleation site density since the size of characteristic radius ξ_1 decreases and approaches the critical radius ξ_{cri} with increasing pressure. In the present analysis, the nucleation site density is assumed to be infinitely large so that the thermal resistance R^* in the drop size range smaller than ξ_1 is taken as a constant at unity. Then the calculated results are considered to give the upper limits for the heat transfer coefficients of dropwise condensation.

Hannemann and Mikic [7] presented a theoretical correlation for steam at atmospheric pressure, which is shown by the dotted curve in Fig. 5. Since their correlation was made using a value of 227 kW m⁻² K⁻¹ as an asymptotic heat transfer coefficient for a surface with infinite thermal conductivity, it agrees well with the experimental data denoted by the open symbols for the surfaces with thermal conductivity higher than around 50 W m⁻¹ K⁻¹. With decreasing thermal conductivity, however, the heat transfer coefficient decreases monotonously below the lower limit estimated by the present theory.

Although we cannot make a close comparison with the experimental data due to the lack of information about the departing drop size, it seems from Fig. 5 that the present theory is capable of describing the constriction resistance in dropwise condensation. Further experimental studies in the reduced pressure range are anticipated to verify the present constriction resistance theory since the heat transfer coefficient is not affected by the difference in the population of microscopic droplets [19].

5. CONCLUSION

The present study derived a fundamental differential equation describing the constriction resistance caused by the nonuniformity of the surface heat flux due to the finite thermal conductivity of the condenser material. The conclusions are as follows:

(1) Heat transfer coefficient of dropwise condensation decreases with decreasing surface thermal conductivity due to increasing constriction resistance.

(2) Constriction resistance can be determined by the Biot number Bi_c and the characteristic non-dimensional drop radius ξ_1 . Bi_c is defined by the interfacial heat transfer coefficient, the departing drop radius and the surface thermal conductivity, and ξ_1 is a function of pressure.

(3) Constriction resistance increases with Bi_c and its increasing rate is significant for the small value of ξ_1 , i.e. for the high pressure condition. Also, at the two limits of Bi_c , the heat transfer coefficient approaches a constant value.

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APPENDIX

In order to obtain an analytical expression for the condensing surface temperature of the adiabatic cylinder system in Fig. 2, the steady heat conduction equation is solved for the following boundary conditions:

$$\begin{pmatrix} \frac{\partial T}{\partial r} \end{pmatrix} = 0 \quad \text{at} \quad r = 0 \quad \text{and} \quad r_{a}$$

$$\begin{pmatrix} \frac{\partial T}{\partial z} \end{pmatrix} = -\frac{Q_{d}}{\lambda_{c}A_{d}} \quad \text{at} \quad z = 0 \quad \text{for} \quad r \leq r_{d}$$

$$\begin{pmatrix} \frac{\partial T}{\partial z} \end{pmatrix} = -\frac{Q_{a}}{\lambda_{c}A_{a}} \quad \text{at} \quad z = 0 \quad \text{for} \quad r_{d} \leq r \leq r_{d}$$

$$T = T_{0} \quad \text{at} \quad z = \delta.$$
(A1)

Here, the surface heat flow rates, Q_d through the central large drop and Q_a through the surrounding area, are expressed as

$$Q_{\rm d} = \frac{T_{\rm s} - (T_{\rm c})_{\rm d}}{R_{\rm d}} A_{\rm d} \tag{A2}$$

$$Q_{a} = \frac{T_{s} - \overline{(T_{c})_{a}}}{R_{a}} A_{a}$$
(A3)

where $(\overline{T_c})_a$ and $(\overline{T_c})_a$ are the averaged surface temperatures for respective parts. From the above boundary conditions, we obtain the following solution for the temperature distribution at the condensing surface (z = 0):

$$T_{\rm c} = \overline{T_{\rm c}} + \frac{2r_{\rm d}}{\lambda_{\rm c}} \left(\frac{Q_{\rm d}}{A_{\rm d}} - \frac{Q_{\rm a}}{A_{\rm a}}\right) \phi(r)$$
$$\phi(r) = \sum_{i=1}^{\infty} \frac{J_{\rm I}(\alpha_i r_{\rm d}/r_{\rm a})}{\alpha_i^2 J_{\rm 0}^2(\alpha_i)} J_{\rm 0}\left(\alpha_i \frac{r}{r_{\rm a}}\right) \tanh\left(\alpha_i \frac{\delta}{r_{\rm a}}\right)$$
(A4)

where J_0 and J_1 are the Bessel functions of zeroth and first order, respectively, and α_i the zeros of $J_1(\alpha_i)$. $\overline{T_c}$ is the surfaceaveraged temperature and is given by

$$\overline{T_{\rm c}} = T_{\rm 0} + \frac{Q_{\rm d} + Q_{\rm a}}{\lambda_{\rm c}(A_{\rm d} + A_{\rm a})}\delta.$$
 (A5)

From Mikic's definition in equation (8), using the weighted mean surface temperature $T_{\rm cm}$, the constriction resistance is expressed as

$$R_{\rm c} = \frac{\frac{4r_{\rm a}}{\lambda_{\rm c}} \left[\frac{1 - R_{\rm a}/R_{\rm d}}{1 - (1 - R_{\rm a}/R_{\rm d})(A_{\rm d}/A)} \right]^2 \frac{A_{\rm d}}{A} \Psi}{1 + \frac{4r_{\rm a}}{\lambda_{\rm c}} \frac{1}{R_{\rm d}} \left(1 + \frac{R_{\rm d}}{R_{\rm a}} \frac{A_{\rm d}}{A_{\rm d}} \right) \Psi}$$
(A6)

$$\Psi = \sum_{i=1}^{\infty} \frac{J_1^2(\alpha_i r_d/r_a)}{\alpha_i^3 J_0^2(\alpha_i)} \tanh\left(\alpha_i \frac{\delta}{r_a}\right)$$
(A7)

where $A = A_d + A_a$. By utilizing Hannemann's approximation [20] for the infinite series

$$\mathbf{I}' = \frac{2}{3\pi} \frac{r_{\rm d}}{r_{\rm a}} \left(1 - \frac{r_{\rm d}}{r_{\rm a}} \right)^{3/2} \tanh\left(4\frac{\delta}{r_{\rm a}}\right) \tag{A8}$$

and considering $r_d/r_a \ll 1$ and $\tanh(4\delta/r_a) \rightarrow 1$, the constriction resistance in this model becomes

$$R_{\rm c} = \frac{8r_{\rm d}}{3\pi\lambda_{\rm c}} \left(1 - \frac{R_{\rm a}}{R_{\rm d}}\right)^2 \left(1 + \frac{8r_{\rm d}}{3\pi\lambda_{\rm c}R_{\rm d}}\right)^{-1} \frac{A_{\rm d}}{A}.$$
 (A9)

By letting $r_d \rightarrow r$, $R_a \rightarrow R$, $R_c \rightarrow R'_c dr$, and utilizing the relation $A_d/A = (1/f)(\partial f/\partial r) dr$, equation (14) is obtained.

UNE ETUDE THEORIQUE DE LA RESISTANCE DE CONSTRICTION DANS LA CONDENSATION EN GOUTTES

Résumé—On étudie théoriquement l'effet de la conductivité thermique du matériau du condenseur sur le transfert thermique pendant la condensation en gouttes. Prenant en compte la contribution de la résistance de gouttelette, une équation aux dérivées partielles fondamentale est donnée pour décrire la résistance de constriction causée par l'hétérogénétié du flux thermique surfacique. On trouve à partie de l'équation fondamentale adimentionnelle que la résistance de constriction peut être déterminée par un nombre de Biot défini par le cefficient de transfert thermique interfacial, le rayon de goutte à la séparation et la conduction thermique de la surface, en plus de quelques paramètres caractéristiques. En appliquant ce qu'on appelle la région d'équilibre des petites gouttes, cette équation es trésolue numériquement et on présente les effets de ces paramètres sur le coefficient de transfert thermique.

THEORETISCHE UNTERSUCHUNG DES MASSGEBLICHEN WIDERSTANDS BEI DER TROPFENKONDENSATION

Zusammenfassung – Der Einfluß der Wärmeleitfähigkeit des Wandmaterials auf den Wärmeübergang bei der Tropfenkondensation wird theoretisch untersucht. Dabei wird eine grundlegende Differentialgleichung abgeleitet, in welcher der Beitrag des Tröpfchenwiderstandes der jeweiligen Größenklasse zum Wärmeübergangswiderstand bei der transienten Tropfenkondensation berücksichtigt wird. Die Differentialgleichung beschreibt den maßgeblichen Widerstand, der durch die ungleichförmige Verteilung der Wärmestromdichte an der Oberfläche verursacht wird. Anhand der in dimensionslose Form gebrachten Gleichung erkennt man, daß der maßgebliche Widerstand mit Hilfe einer Biot-Zahl ermittelt werden kann. Diese wird mit dem Wärmeübergangskoeffizienten an der Phasengrenze, dem Abreißdurchmesser der Tröpfchen, der Wärmeleitfähigkeit, des Oberflächenmaterials und einigen charakteristischen Parametern gebildet. Um den Einfluß dieser Größen auf den Wärmeübergangskoeffizienten darzustellen, wird diese Gleichung numerisch unter Verwendung des sogenannten Gleichgewichtsgebiets kleiner Tropfen als Tropfengrößenverteilung gelöst.

ТЕОРЕТИЧЕСКОЕ ИССЛЕДОВАНИЕ СОПРОТИВЛЕНИЯ СТЯГИВАНИЮ ПРИ КАПЕЛЬНОЙ КОНДЕНСАЦИИ

Аннотация—Теоретически исследуется влияние теплопроводности материала конденсатора на теплоперенос при капельной конденсации. С учетом вклада сопротивления капель для конкретного диапазона их размеров в термическое сопротивление при нестационароной капельной конденсации выводится дифференциальное уравнение, описывающее сопротивление стягиванию, вызванное неоднородностью теплового потока на поверхности. Из этого уравнения, записанного в безразмерном виде, найдено, что сопротивление стягиванию может быть рассчитано с помощью числа Био, которое определяется коэффициентом теплопереноса на границе раздела, радиусом отрывающейся капли, коэффициентом теплопроводности поверхности и некоторыми характерными параметрами. Для установления влияния указанных величин на коэффициент теплопереноса данное уравнение решается численно с использованием равновесного распределения по размерам в области малых капель.